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Entropy in Kerr–de Sitter Black Holes With Spin Fields

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Generalized master equations due to spin fields are given. We obtain the entropy of electromagnetic, gravitational, Dirac, and scalar fields in a unified form by using the improved brick-wall method—membrane model. The results show that, as the cutoff is properly chosen, the entropy in the black hole satisfies the Bekenstein–Hawking area formula.

KEY WORDS: entropy; Kerr–de Sitter black holes; spin fields.

1. INTRODUCTION

Since Bekenstein and Hawking initiated the discussion on the entropy of a black hole in 1979s, more efforts have been devoted to studying the statistical origin of the black holes' entropy. 't Hooft proposed brick-wall method and studied the statistical mechanics of a free scalar field in the Schwarzschild black hole background in 1985 ('t Hooft, 1985), and found that the entropy of the scalar field is proportional to area of the black hole horizon, but the entropy is divergent as the cutoff is taken to be zero. However, he thought the divergence is caused by the infinite density of states approaching the horizon. In the middle of 1990s, series of work has been done in this respect (Demers *et al.*, 1995; Ghosh and Mitra, 1994, 1995; Lee and Kim, 1996). Recently, the membrane model was developed basing on the brick-wall method (Gao and Shen, 2001; Li and Zhao, 2000). The entropy of various black holes has been calculated through the above methods and many

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important consequences have been achieved (Chandrasekhar, 1983; Gao and Shen, 2002; Liberati and Pollofrone, 1997; Shen, 2000, 2002; Shen *et al.*, 1997).

However, the entropy of a black hole which is not spherically symmetric in electromagnetic ($s = 1$) and gravitational ($s = 2$) fields has not yet been studied. In this paper, we calculated the entropy of electromagnetic, gravitational, Dirac, and scalar fields in the Kerr–de Sitter black holes which is not spherically symmetric. Considering there are two event horizons in the Kerr–de Sitter black hole: black hole event horizon and cosmos event horizon, we propose to calculate the entropy of the black holes by using the improved brick-wall method—membrane model rather than by purely brick-wall method which would make our calculation complex. The radiation between the two horizons is not in thermal equilibrium because of their different temperatures. Thus, we regard the horizons as two independent thermal equilibrium systems and the entropy refer to black hole can be calculated respectively. The results show that as the cutoff is properly chosen, the entropy in the black hole satisfies Bekenstein–Hawking area formula.

2. KERR–de SITTER METRIC

The Kerr–de Sitter space-times can be written in Boyer–Lindquist coordinates as (Ahmed, 1991)

$$\begin{aligned} ds^2 = {} & \frac{\Delta_r - \Delta_\theta a^2 \sin^2\theta}{\rho^2 J^2} dt^2 + 2\frac{a\sin^2\theta[\Delta_\theta(r^2+a^2) - \Delta_r]}{\rho^2 J^2} dt\, d\phi \\ & - \frac{\rho^2}{\Delta_r} dr^2 - \frac{\rho^2}{\Delta_\theta} d\theta^2 - \frac{\Delta_\theta(r^2+a^2)^2 - \Delta_r a^2 \sin^2\theta}{\rho^2 J^2} d\phi^2, \end{aligned} \tag{1}$$

where ρ^2, $\bar{\rho}$, Δ_r, Δ_θ, J^2 are defined by

$$\begin{aligned} \rho^2 &= \bar{\rho} \cdot \bar{\rho}^*, \quad \bar{\rho} = r + ia\cos\theta, \\ \Delta_r &= (r^2 + a^2)\left(1 - \frac{1}{3}\Lambda r^2\right) - 2Mr, \\ \Delta_\theta &= 1 + \frac{1}{3}\Lambda a^2 \cos^2\theta, \\ J &= 1 + \frac{1}{3}\Lambda a^2, \end{aligned} \tag{2}$$

where M, a, Λ are the mass, angular momentum per unit mass, and the cosmological constant. The null vectors of the Newman–Penrose formalism we take

$$\begin{aligned} l^\mu &= \left[\frac{J(r^2+a^2)}{\Delta_r}, 1, 0, \frac{aJ}{\Delta_r}\right], \\ n^\mu &= \frac{1}{2\rho^2}[J(r^2+a^2), -\Delta_r, 0, aJ], \end{aligned}$$

$$m^{\mu} = \frac{1}{\sqrt{2\Delta_\theta}\bar{\rho}}\left[iaJ\ \sin\theta, 0, \Delta_\theta, \frac{iJ}{\sin\theta}\right]. \tag{3}$$

We find the nonvanishing spin-coefficients listed below

$$\pi = \frac{ia\sqrt{\Delta_\theta}\sin\theta}{\sqrt{2}\bar{\rho}^{*2}};\ \mu = -\frac{\Delta_r}{2\rho^2\bar{\rho}^*};\ \alpha = \pi - \beta^*;\ \beta = \frac{1}{2\sqrt{2}\bar{\rho}\sin\theta}\frac{d(\sqrt{\Delta_\theta}\sin\theta)}{d\theta};$$

$$\tau = -\frac{ia\sqrt{\Delta_\theta}\sin\theta}{\sqrt{2}\rho^2};\ \rho = -\frac{1}{\bar{\rho}^*};\ \gamma = \frac{1}{4\rho^2}\frac{d\Delta_r}{dr} + \mu. \tag{4}$$

Assuming that the azimuthal and time dependence of our fields will be of the form $e^{i(m\phi-\omega t)}$, we find that the directional derivatives are

$$D = l^\mu\partial_\mu = D_0, \quad \Delta = n^\mu\partial_\mu = \frac{-\Delta_r}{2\rho^2}D_0^+,$$

$$\delta = m^\mu\partial_\mu = \frac{\sqrt{\Delta_\theta}}{\bar{\rho}\sqrt{2}}L_0^+, \quad \delta^* = m^{*\mu}\partial_\mu = \frac{\sqrt{\Delta_\theta}}{\bar{\rho}^*\sqrt{2}}L_0, \tag{5}$$

where D_n, D_n^+, L_n, L_n^+, K, H are defined by

$$\begin{aligned}
D_n &= \partial_r + \frac{iJK}{\Delta_r} + \frac{n}{\Delta_r}\frac{d\Delta_r}{dr},\\
D_n^+ &= \partial_r - \frac{iJK}{\Delta_r} + \frac{n}{\Delta_r}\frac{d\Delta_r}{dr},\\
L_n &= \partial_\theta + \frac{JH}{\Delta_\theta} + \frac{n}{\sqrt{\Delta_\theta}\sin\theta}\frac{d(\sqrt{\Delta_\theta}\sin\theta)}{d\theta},\\
L_n^+ &= \partial_\theta - \frac{JH}{\Delta_\theta} + \frac{n}{\sqrt{\Delta_\theta}\sin\theta}\frac{d(\sqrt{\Delta_\theta}\sin\theta)}{d\theta},\\
K &= am - \omega(r^2 + a^2),\\
H &= \frac{m}{\sin\theta} - a\ \sin\theta
\end{aligned} \tag{6}$$

Thus K and H have the relation

$$K - aH\ \sin\theta = -\rho^2\omega. \tag{7}$$

These differential operators satisfy some identities

$$\Delta_r D_{n+1} = D_n\Delta_r, \tag{8}$$

$$\Delta_r D_{n+1}^+ = D_n^+\Delta_r, \tag{9}$$

$$(\sqrt{\Delta_\theta}\sin\theta)L_{n+1} = L_n\sqrt{\Delta_\theta}\sin\theta, \tag{10}$$

$$(\sqrt{\Delta_\theta}\sin\theta)L^+_{n+1} = L^+_n\sqrt{\Delta_\theta}\sin\theta, \tag{11}$$

$$\left(D+\frac{m}{\bar{\rho}^*}\right)\sqrt{\Delta_\theta}\left(L+\frac{ima\,\sin\theta}{\bar{\rho}^*}\right) = \sqrt{\Delta_\theta}\left(L+\frac{ima\,\sin\theta}{\bar{\rho}^*}\right)\left(D+\frac{m}{\bar{\rho}^*}\right). \tag{12}$$

3. SPIN FIELDS KERR–de SITTER SPACE TIME

The Maxwell equations in the Newman–Penrose formalism take on the forms

$$D\phi_1 - \delta^*\phi_0 = (\pi - 2\alpha)\,\phi_0 + 2\tilde{\rho}\phi_1 - \kappa\phi_2, \tag{13}$$

$$D\phi_2 - \delta^*\phi_1 = -\lambda\phi_0 + 2\pi\phi_1 + (\tilde{\rho} - 2\epsilon)\phi_2, \tag{14}$$

$$\delta\phi_1 - \Delta\phi_0 = (\mu - 2\gamma)\,\phi_0 + 2\tau\phi_1 - \sigma\phi_2, \tag{15}$$

$$\delta\phi_1 - \Delta\phi_0 = -\nu\phi_0 + 2\mu\phi_1 + (\tau - 2\beta)\,\phi_2. \tag{16}$$

Using Eqs. (4) and (5), then making the transformations

$$\phi_0 = \Phi_0, \quad \phi_1 = \frac{1}{\sqrt{2}\bar{\rho}^*}\Phi_1$$

and

$$\phi_2 = \frac{1}{(2\bar{\rho}^*)^2}\Phi_1,$$

we can separate Eqs. (l3)–(16) to be (Khanal, 1983)

$$(\Delta D_1 D_1^+ - 2iJr\omega)\,R_{+1} = \lambda R_{+1}, \tag{17}$$

$$(\Delta D_0^+ D_0 + 2iJr\omega)\,R_{-1} = \lambda R_{-1}, \tag{18}$$

$$[\sqrt{\Delta_\theta}L_0^+\sqrt{\Delta_\theta}L_1 + 2J\omega a\,\cos\theta]\,S_{+1} = -\lambda S_{+1}, \tag{19}$$

$$[\sqrt{\Delta_\theta}L_0\sqrt{\Delta_\theta}L_1^+ - 2J\omega a\,\cos\theta]\,S_{-1} = -\lambda S_{-1}, \tag{20}$$

here λ is the separation constant. Multiplying Eq. (17) by Δ_r, and using the conditions (8) and (9), we can rewrite Eq. (17) as

$$\left(\Delta_r D_0 D_0^+ - 2iJr\omega\right)\Delta_r R_{+1} = \lambda\Delta_r R_{+1}. \tag{21}$$

For the Dirac field, the wave equation for a massless dirac particle is (Khanal, 1983)

$$(D+\varepsilon-\rho)\,F_1 + (\bar{\delta}+\pi-\alpha)\,F_2 = 0,$$

$$(\Delta' - \mu - \gamma)\,F_2 + (\delta+\beta-\tau)\,F_1 = 0,$$

$$(D+\varepsilon^*-\rho^*)\,G_2 - (\delta+\pi^*-\alpha^*)\,G_1 = 0,$$

$$(\Delta' - \mu^* - \gamma^*)\,G_1 - (\bar{\delta}+\beta^*-\tau^*)\,G_2 = 0, \tag{22}$$

where F_1, F_2, G_1, G_2 are four-component spinors α, β, γ, ϵ, μ, π, ρ, τ, etc. are Newman–Penrose symbols, and α^*, β^* are the complex conjugates of α, β, etc.

All the above equations are also separated by using Newman–Penrose formalism. The radial equations are given by

$$\Delta^{\frac{1}{2}} D_0^+ \left(\Delta^{\frac{1}{2}} D_0 R_{-\frac{1}{2}}\right) = \lambda^2 R_{-\frac{1}{2}}, \tag{23}$$

$$\Delta^{\frac{1}{2}} D_0 \left(\Delta^{\frac{1}{2}} D_0^+ R_{-\frac{1}{2}}\right) = \lambda^2 R_{+\frac{1}{2}}, \tag{24}$$

$$L_{\frac{1}{2}}^+ \left(L_{\frac{1}{2}} S_{+\frac{1}{2}}\right) = -\lambda^2 S_{+\frac{1}{2}}, \tag{25}$$

$$L_{\frac{1}{2}} \left(L_{\frac{1}{2}}^+ S_{-\frac{1}{2}}\right) = -\lambda^2 S_{-\frac{1}{2}}. \tag{26}$$

For the gravitational field, we find that the Weyl scalars ψ_0, ψ_1, ψ_3, and ψ_4 vanish (leaving $\psi_2 = -M/(\bar{\rho}^*)^3$) along with the spin coefficients κ, σ, λ, and ν, showing that the metric is of type D. Then equations for the perturbed Weyl scalars can be written as

$$\begin{aligned}
(\delta^* - 4\alpha + \pi)\,\psi_0 - (D - 2\epsilon - 4\tilde{\rho})\,\psi_1 &= 3\kappa\psi_2, \\
(\Delta - 4\gamma + \mu)\,\psi_0 - (\delta - 2\beta - 4\tau)\,\psi_1 &= 3\sigma\psi_2, \\
(D - \tilde{\rho} - \tilde{\rho}^* - 3\epsilon + \epsilon^*)\,\sigma - (\delta - \tau + \pi^* - \alpha^* - 3\beta)\,\kappa &= \psi_0; \\
(D - 4\epsilon - \tilde{\rho})\psi_4 - (\delta^* + 4\pi + 2\alpha)\,\psi_3 &= -3\lambda\psi_2, \\
(\delta + 4\beta - \tau)\,\psi_4 - (\Delta + 2\gamma + 4\mu)\,\psi_3 &= -3\nu\psi_2, \\
(\Delta + \mu + \mu^* + 3\gamma - \gamma^*)\,\lambda - (\delta^* + 3\alpha + \beta^* + \pi - \tau^*)\,\nu &= -\psi_4.
\end{aligned} \tag{27}$$

Making the transformations

$$\begin{aligned}
\psi_0 &= \Phi_0, \\
\psi_1 &= \frac{1}{\sqrt{2}\bar{\rho}^*}\Phi_1, \quad \psi_3 = \frac{\sqrt{2}}{(\bar{\rho}^*)}, \\
\psi_4 &= \frac{1}{(\bar{\rho}^*)^3}\Phi_4,
\end{aligned} \tag{28}$$

Equation (27) can be finally separated into (Khanal, 1983)

$$(\Delta_r D_1 D_2^+ - 6iJ\omega r - 2\Lambda r^2)\Phi_0 = \lambda R_{+2}, \tag{29}$$

$$(\sqrt{\Delta_\theta} L_{-1}^+ \sqrt{\Delta_\theta} L_2 + 6aJ\omega\cos\theta - 2\Lambda a^2\cos\theta^2) S_{+2} = -\lambda S_{+2}, \tag{30}$$

$$(\Delta_r D_{-1}^+ D_0 + 6iJ\omega r - 2\Lambda r^2) R_{-2} = \lambda R_{-2}, \tag{31}$$

$$(\sqrt{\Delta_\theta} L_{-1} \sqrt{\Delta_\theta} L_2^+ - 6aJ\omega\cos\theta - 2\Lambda a^2\cos\theta^2) S_{-2} = -\lambda S_{-2}. \tag{32}$$

We can also see that Eq. (29) can be multiplied by Δ_r^2 to give

$$(\Delta_r D_{-1} D_0^+ - 6iJr\omega - 2r^2\Lambda)\Delta_r^2 R_{+2} = \lambda \Delta_r^2 R_{+2}, \tag{33}$$

for scalar field, the separated equations can achieved directly from the Klein–Gordon equation

$$\frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^\mu} \left(\sqrt{-g} g^{\mu\nu} \frac{\partial \Phi}{\partial x^\nu} \right) = 0. \tag{34}$$

The radial equation is

$$\frac{\partial}{\partial r} \Delta \frac{\partial}{\partial r} R(r) + \frac{[(r^2 + a^2)\omega - am]^2}{\Delta} R(r) = \lambda^2 R(r). \tag{35}$$

We can rewrite the above equation with D_n, D_n^+

$$\Delta D_0 D_0^+ R_0 = \lambda^2 R_0 \tag{36}$$

or

$$\Delta D_0^+ D_0 R_{-0} = \lambda^2 R_{-0}. \tag{37}$$

The radial equations (21), (24), (31), and (36) can be combined into

$$\left[\Delta_r D_{\frac{1}{2}s(8s^2-7s+5)} D_0^+ - s(s-3)(1-2s)\, i\omega r - \frac{\Lambda}{11} r^2 s(s-1)(6s-1) \right] \times \Delta_r^s R_s = \lambda^2 \Delta_r^s R_s. \tag{38}$$

On the other hand, Eqs. (18), (23), (31), and (37) can be written as

$$\left[\Delta_r D^+_{\frac{1}{2}s(8s^2-7s+5)} D_0 - s(s-3)(1-2s)\, i\omega r - \frac{\Lambda}{11} r^2 s(s-1)(6s-1) \right] R_{-s} = \lambda^2 R_{-s}, \tag{39}$$

where s is the spin number. It is clear that as $s = 0, s = \frac{1}{2}, s = 1,$ and $s = 2$, Eq. (35) correspond to scalar, Dirac, Maxwell, and gravitational field, respectively.

4. ENTROPY

By using the Wentzel–Kramers–Brillouin approximation and substituting equations $R_s = e^{if(r)}$ into Eq. (35), the wave numbers are obtained as follows,

$$k_r^2 = \frac{K^2 J^2}{\Delta_r^2} + \frac{s}{\Delta_r} \frac{d^2 \Delta_r}{dr^2} + \frac{\frac{1}{2}s^2(8s^2 - 7s + 5) + s^2 - s}{\Delta_r^2} \left(\frac{d\Delta}{dr} \right)^2 - \frac{\lambda^2}{\Delta_r} - \frac{\Lambda}{11\Delta_r} r^2 s(s-1)(6s-1). \tag{40}$$

Considering while $s = 0, \frac{1}{2}, 1$, and 2, the third term of the equation is zero, we remove it from our equation. Therefore we have

$$k_r = \frac{1}{\Delta_r}\sqrt{J^2[(r^2+a^2)\omega - am]^2 + 2\Delta_r s\left(1 - 4\pi r^2 - \frac{1}{3}\Lambda a^2\right)} \times\sqrt{-\frac{\Lambda}{11}\Delta_r r^2 s(s-1)(6s-1) - \Delta_r\lambda^2}. \tag{41}$$

The horizon equation can be written as

$$\Delta_r = -\frac{\Lambda}{3}(r - r_+)(r - r_{++})(r - r_-)(r - r_{--}) = 0, \tag{42}$$

where r_+, r_{++}, r_-, r_{--} are the radius of the black hole event horizon, cosmos event horizon, black hole Cauchy horizon, and the cosmos Cauchy horizon, respectively. We assume that the fields discussed are in the Hartle–Hawking vacuum state, thus the angular velocity, the Hawking temperature of the black hole event horizon, and the cosmos event horizon are defined by

$$\Omega_+ = \frac{a}{r_+^2 + a^2}, \quad \Omega_{++} = \frac{a}{r_{++}^2 + a^2}, \tag{43}$$

$$T_+ = \frac{1}{\beta_+} = -\frac{\Lambda(r_+ - r_{++})(r_+ - r_-)(r_+ - r_{--})}{12\pi J(r_+^2 + a^2)}, \quad T_{++} = \frac{1}{\beta_{++}} = -\frac{\Lambda(r_{++} - r_+)(r_{++} - r_-)(r_{++} - r_{--})}{12\pi J(r_{++}^2 + a^2)}. \tag{44}$$

In order to be easy to calculate, we set $E = \omega - m\Omega_+$, thus the wave numbers refer to the black hole event horizon can be written as

$$k_r = \frac{1}{\Delta_r} \times \sqrt{J^2(r^2+a^2)(E + m\Omega_+ - m\Omega)^2 + 2\Delta_r s\left(1 - 4\Lambda r^2 - \frac{1}{3}\Lambda a^2\right)} \times\sqrt{-\frac{\Lambda}{11}\Delta_r r^2 s(s-1)(6s-1) - \Delta_r\lambda^2}, \tag{45}$$

where $\Omega = \frac{a}{r^2+a^2}$. When we substitute Ω_+ for Ω_{++} in the above equation, we can get the wave numbers refer to the cosmos event horizon. It's clear that the expression of the wave numbers is very complex. However, we will see that the second and third terms do not contribute to the calculation of the free energy and entropy. The free energy at temperature T_+ of the boson system is given by

$$\beta_+ f_+ = -\sum_E \ln(1 \pm e^{-\beta_+ E}), \tag{46}$$

where $+$ corresponds to fermion field and $-$ corresponds to boson field.

According to semiclassical quantum theory, there is

$$\sum_E \to \int_0^\infty dEg(E),$$

where $g(E) = \omega' \frac{d\Gamma(E)}{dE}$ is the states density; ω' is the degeneracy of the fields (for scalar field and neutrino field, $\omega' = 1$ for Maxwell and gravitational field, $\omega' = 2$). The states number is

$$\Gamma(E) = \sum_{m,l} n_r(E, l, m) = \int dm \int dl \frac{1}{\pi} \int k_r(E, l)\, dr. \tag{47}$$

The free energy can be calculated as follows

$$\begin{aligned}
-\beta_+ f_+ &= \pm \int_0^\infty dEg(E) \ln(1 \pm e^{-\beta_+ E}) \\
&= \pm\beta_+ \int_0^\infty dE\omega' \frac{\Gamma(E)}{e^{\beta_+ E} \pm 1} \\
&= \frac{\beta_+\omega'}{\pi} \int_0^\infty dE \int_{r_+ + \varepsilon}^{r_+ + 2\varepsilon} dr \int_0^{\lambda_{\max}} d\lambda \int_{-\lambda}^{\lambda} dm \frac{1}{\Delta_r} (e^{\beta_+ E} \pm 1)^{-1} \\
&\quad \times \sqrt{J^2(r^2 + a^2)(E + m\Omega_+ - m\Omega)^2 + 2\Delta_r s \left(1 - 4\Lambda r^2 - \frac{1}{3}\Lambda a^2\right)} \\
&\quad \times \sqrt{-\frac{\Lambda\Delta_r}{11} r^2 s(s-1)(6s-1) - \Delta_r \lambda^2}.
\end{aligned} \tag{48}$$

Considering fermions field and bosons field, the results can be written as

$$f_{+f} = -\frac{7}{20} \frac{\pi^3}{\beta^3} \frac{J^3(r^2 + a^2)^3}{\Lambda^2(r_+ - r_{++})^2(r_+ - r_-)^2(r_+ - r_{--})^2} \frac{\varepsilon}{\eta^2}, \text{ (fermions field)} \tag{49}$$

$$f_{+b} = -\frac{2}{5} \frac{\pi^3}{\beta^3} \frac{J^3(r^2 + a^2)^3}{\Lambda^2(r_+ - r_{++})^2(r_+ - r_-)^2(r_+ - r_{--})^2} \frac{\varepsilon}{\eta^2}, \text{ (bosons field)} \tag{50}$$

where ε is the ultraviolet regulator, which satisfies $0 < \varepsilon \ll r_+$. This manifests that the integral over the quantum number m does not diverge, therefore we need not to regularize the m integral. On the other hand, the membrane model illustrates that the black hole entropy mainly comes from the vicinity of event horizon. Thus we have taken into account the following equation in the integration with respect to m,

$$\lim_{r \to r_+} \Omega = \Omega_+. \tag{51}$$

We also used the median theorem in the integration with respect to r, hence $\varepsilon < \eta < 2\varepsilon$. The extreme of integration in the variable l is due to the fact that k_r, has to be positive. Eq. (39) is also used in the integration with respect to r.

We obtain the entropy due to an arbitrary spin field of the Kerr–de Sitter black hole from the standard formula

$$S = \beta^2 \frac{\partial F}{\partial \beta}. \tag{52}$$

As to fermion field, one componential entropy can be written as

$$S_{1f} = \frac{7}{5} \frac{\pi^3}{\beta^3} \frac{J^3(r^2 + a^2)^3 \omega'}{\Lambda^2 (r_+ - r_{++})^2 (r_+ - r_-)^2 (r_+ - r_{--})^2} \frac{\varepsilon}{\eta^2}. \tag{53}$$

There are four components of the wave function refer to fermion field. Therefore the whole black hole entropy is given by

$$S_{+f} = 4S_1 = \frac{28}{5} \frac{\pi^3}{\beta^3} \frac{J^3(r^2 + a^2)^3 \omega'}{\Lambda^2 (r_+ - r_{++})^2 (r_+ - r_-)^2 (r_+ - r_{--})^2} \frac{\varepsilon}{\eta^2}. \tag{54}$$

Similarly, the entropy of bosons field can be obtained as

$$S_{+b} = \frac{8}{5} \frac{\pi^3}{\beta^3} \frac{J^3(r^2 + a^2)^3 \omega'}{\Lambda^2 (r_+ - r_{++})^2 (r_+ - r_-)^2 (r_+ - r_{--})^2} \frac{\varepsilon}{\eta^2}. \tag{55}$$

We choose the cutoff as $\frac{1}{\varepsilon} = 90\beta / J\omega'$. Here ε and η in Eqs. (47) and (48) are of the same order. Therefore $\frac{\varepsilon}{\eta^2} \sim \frac{1}{\varepsilon} = 90\beta / J\omega'$, then the entropy in the Eqs. (47) and (48) satisfies the area law

$$S_{+f} = \frac{7}{8} 4\pi J^{-1} (r_+^2 + a^2) = \frac{7}{8} A_+, \tag{56}$$

$$S_{+b} = \frac{1}{4} 4\pi J^{-1} (r_+^2 + a^2) = \frac{1}{4} A_+, \tag{57}$$

where A_+ is the area of black hole event horizon. In the same way, we can easily obtain the expected entropy of the cosmos event horizon, i.e.,

$$S_{++f} = \frac{7}{8} 4\pi J^{-1} (r_{++}^2 + a^2) = \frac{7}{8} A_{++}, \tag{58}$$

$$S_{++b} = \frac{1}{4} 4\pi J^{-1} (r_{++}^2 + a^2) = \frac{1}{4} A_{++}, \tag{59}$$

where A_{++} is the area of cosmos event horizon. Thus the whole entropy of the Kerr–de Sitter black hole is given by

$$S_f = S_{+f} + S_{++f} = \frac{7}{8} (A_+ + A_{++}), \tag{60}$$

$$S_b = S_{+b} + S_{++b} = \frac{1}{4} (A_+ + A_{++}). \tag{61}$$

5. CONCLUSION

We have studied the entropy due to spin fields in the Kerr–de Sitter black holes by using the membrane model. And also unified radar equations of four fields are given. Our results may be significant because they are for black holes that are not spherically symmetric and especially their entropy of their Maxwell field and gravitational field is rarely calculated. Since the cutoff was properly chosen, the Kerr–de Sitter black hole entropy is identified with the Bekenstein–Hawking area formula. We can see from the results that the electromagnetic, Dirac, gravitational, and scalar field entropies of the following black holes: Schwarzschild black hole, Reissner–Nordström black hole, Kerr black hole, are embodied as special cases of the Kerr–de Sitter black hole entropy.

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